American University of Beirut
Department of Computer Science
CMPS 211 – Discrete Mathematics – Fall 14/15

Please solve the following exercises and submit **BEFORE 8:00 am of Tuesday 14th, October.**

Exercise 1 (15 points)

a) Prove that if a + b and b + c are odd integers where a, b and c are integers, then a + c is even. What kind of proof did you use?
b) Prove that if a and b are integers and ab is odd then both of a and b are odd. What kind of proof did you use?

Exercise 2 (15 points)

a) Prove by contradiction that the sum of an irrational number and a rational number is irrational.
b) Prove or disprove that the product of a nonzero rational number and an irrational number is irrational.

Exercise 3 (10 points)

Prove that the proposition P(1), where P(n) is the proposition "If n is a positive integer then $2n \geq n+1$" is true. What kind of proof did you use?

Exercise 4 (10 points)

Prove that if n is a perfect cube, then n+3 is not a perfect cube.

Exercise 5 (10 points)

Prove that if n is a positive integer, then n is odd if and only if 3n + 2 is odd.

Exercise 6 (10 points)

Is this reasoning for finding the solutions of the equation $\sqrt{(5x^2 - 4)}=2x$ correct? *(1)* $\sqrt{(5x^2-4)}=2x$ is given; *(2)* $5x^2 - 4 = 4x^2$, obtained by squaring both sides of (1); *(3)* $x^2 - 4 = 0$, obtained by subtracting $4x^2$ from both sides of (2); *(4)* $(x - 2)(x + 2) = 0$, obtained by factoring the left-hand side of x^2-4; *(5)* $x=2$ or $x=-2$, which follows because $ab = 0$ implies that $a = 0$ or $b=0$.

Exercise 7 (10 points)

Prove that there is a positive integer that equals the sum of the positive integers not exceeding it. Is your proof constructive or non-constructive?

Exercise 8 (10 points)

Show that these statements about the real number x are equivalent:

1. x is rational,
2. x/3 is rational,
3. 5x – 2 is rational.

Exercise 9 (10 points)

Show that these statements about the integer n are equivalent:

1. n^2 is even,
2. 1-n is odd,
3. n^3 is even,
4. $n^2+ 1$ is odd.